

Then Eq. (8) shows that

$$r' = 2 + \frac{1}{S^2} - \frac{\text{erf}(S)}{8S^4(Sr')} \quad (9)$$

Equations (8) and (9) seem particularly simple to use in Eqs. (4) and (1) to give the heat transfer rate to spheres in free molecule flow.

Two other references have recently come to my attention with errors in the recovery factor. One is V. P. Shidlovskiy, *Introduction to Dynamics of Rarefied Gases*, American Elsevier, New York, 1967. On page 41, Eq. (2.52), the factor in front of erfs in the numerator should be divided by 2. The second is S. A. Schaaf, "Mechanics of Rarefied Gases," in *Handbuch Der Physik*, Vol. III, No. 2, Springer-Verlag, Berlin, 1963. On page 605, Eq. (7.6), the last factor in the denominator should be multiplied by  $S^2$ .

#### Acknowledgment

The authors' colleague G. E. Caledonia pointed out the error in  $r'$  in Ref. 4 and suggested writing this Comment.

#### References

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- <sup>2</sup>Schaaf, S. A. and Talbot, L., "Handbook of Supersonic Aerodynamics," Navord Rept. 1488, Feb. 1959.
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- <sup>4</sup>Schaaf, S. A., "Heat Transfer in Rarefied Gases," *Developments in Heat Transfer*, edited by W. M. Rohsenow, The M.I.T. Press, Cambridge, Mass., 1964, Chap. 7.
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- <sup>8</sup>Stalder, J. and Zurick, V., "Theoretical Characteristics of Bodies in a Free Molecule Flow Field," NASA TN 2423, 1951.
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### Reply by Author to N.H. Kemp

Dennis R. Hall\*

Massachusetts Institute of Technology,  
Lexington, Mass.

THE author is happy that his Technical Note<sup>1</sup> has inspired Dr. Kemp to correct many erroneous printings of the free-molecular heating and drag equations which were published and republished by many people over the last two decades. The error in the recovery factor does, as he implied, have negligible effect on the predicted temperatures of Ref. 1. In particular, for a speed ratio of 13, the recovery used there

is  $\approx 0.2\%$  higher than it would be using the correct recovery factor equation of Dr. Kemp. As  $s$  becomes bigger the error becomes smaller. On the other hand, if the tests were conducted near speed ratios of 2 to 4 the author would have made obvious temperature mispredictions.

#### References

- <sup>1</sup>Hall, D. R., "Hypersonic Free-Molecular Heating of Micron Size Particulate," *AIAA Journal*, Vol. 16, Aug. 1978, pp. 857-859.

### Rebuttal to Reply by Author to "Comment on 'Flutter of Flat Finite Element Panels in a Supersonic Potential Flow' "

William P. Rodden\*  
La Cañada Flintridge, Calif.

DR. Yang's criticisms in his Reply<sup>1</sup> to my Comment<sup>2</sup> on his paper<sup>3</sup> were primarily limited to the structural aspects of the problem, and he replied to points most of which were conceded in the Comment. The matter of aerodynamic influence coefficients (AICs), however, deserves a few additional remarks. The AICs, as Revell and I have defined them,<sup>4</sup> constitute an aerodynamic finite element counterpart to the structural displacement method since they both relate forces to displacements. Nelson and Cunningham's work<sup>5</sup> certainly was published before my thesis, but they solved the stability problem by a routine application of the Galerkin method. By 1956, however, the validity of the Galerkin method had been called into question by a number of investigators because of an apparent paradox in analyzing the membrane panel (i.e., a panel with no bending stiffness) which is stable. Hence, the novelty of my thesis was found in a completely different approach, i.e., using both the structural and aerodynamic influence coefficients, which demonstrated the correctness of the Nelson and Cunningham Galerkin solution and contributed to a better understanding of the Galerkin method. Dr. Yang would not concede that my two-dimensional panel AICs preceded his (by 20 years) or that they were based on a more efficient computational algorithm. The point of referencing my thesis, or better yet, the *AGARD Manual on Aeroelasticity*<sup>6</sup> which discussed my AICs, is that this is where the first use of AICs appears in panel flutter analysis and the AICs included the *second order* frequency term.

Dr. Yang depreciates my treatment of the second order term when he replies: "When the simple trapezoidal rule can achieve excellent accuracy in approximating the aerodynamic pressure, the use of sophisticated higher-order numerical integration method is of no value, especially when the structural model is so crude." The remark on the crude structural model is, of course, irrelevant to any aerodynamic discussion, but Dr. Yang does not recognize the fact that he did not simply use a trapezoidal rule. He used the trapezoidal rule and then an *additional averaging* between grid points that increased his accuracy to be comparable to my "sophisticated" integration, but at some computational

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\*Engineer, Aerodynamics, Aerospace Division, Lincoln Laboratory.

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\*Consulting Engineer. Associate Fellow AIAA.